CSMA MULTIHOP NETWORKS
Throughput Analysis

by Dr. Bob W. McGwier N4HY

Amateur Radio uses packet radio in a broadcast mode and typically uses omnidirectional antennas for transmission. If the two ends of the data transfer are not within communication range, we will use a digipeater. The protocol we use for medium access control is carrier sensed multiple access (CSMA) protocol. Under CSMA, a terminal will not transmit if it hears a transmission in its neighborhood. In a multihop environment (digipeater) such as ours, CSMA is subject to "hidden transmitter" interference. This is when two or more transmitters outside hearing range of each other key up at the same time and interfere with one or more recipients of the two or more packets being simultaneously transmitted. We call this a collision. We could almost completely eliminate this problem with busy tone multiple access (BTMA) protocol. The "almost completely" in the previous sentence is caused by the finite speed of light and the response time of the receiver bringing up the busy tone. BTMA has implicit in its nature seperate transmit and receive operation. Several papers have appeared analyzing throughput in CSMA packet radio networks. The best papers and the most useful (not always the same) are by Boorstyn and Kershenbaum and we include one of their papers as our only reference as it contains the most complete set of references and is the basis for the work described here. They have introduced a continuous time Markov Chain model which lends itself to numerical techniques for finding the steady state response of moderate sized (100-200 nodes) networks. This type of model allows for dependencies between non-adjacent nodes to be modeled and analyzed. The primary purpose of this paper is to introduce these ideas to the amateur radio literature, to supply some rigor to Clark's "But Wait There's More" treatise, and hopefully give one more nudge to do something about the problem.

Our Packet Networks and CSMA

Our networks are made up of terminal node controllers (hereinafter TNC or simply node) with radios for broadcasting packetized data over limited distances (forget HF, that problem is enormously complicated). In the most general case, the source and destination nodes will not be able to hear each other directly, and will digipeat through intermediate TNC's. We assume that the network topology and traffic requirements operate on a time scale sufficiently long to establish steady state conditions. We assume an idealized AX25L2V2 model and completely neglect COSI, IP, NET/ROM, or TEXNET and any point by point acks or dynamic routing these systems may employ.

The Packet) Radio Markov Model

We make a restrictive model, which we hope will include only the effects of CSMA and not the 2211 modems we all use. Our model is

1) Nodes schedule transmissions to neighbors, that is they can hear each other according to a independent Poisson point processes. The arrival times of crashes on 75 meters on a summer night is modeled by a Poisson point process quite well.

2) Packet lengths are exponentially distributed and are generated independently at each transmission. This can be greatly relaxed but we will not do so here.

3) The propagation delay between adjacent nodes is zero.

4) Under CSMA, we will begin our scheduled transmission if we are neither sending or receiving or hearing a packet.

5) Nodes receive with perfect capture. A bad assumption that allows us to do the mathematics that follows. This means if A transmits to B and then during this transmission a neighbor of B, node C, begins to transmit, node B still receives A's transmission. Thus the collision only happens if the recipient of C's packet can hear the A node transmission. In this case, C must retransmit.

6) Links are error free (2211's and MF-10's and OUR radios????). Again this assumption is to allow ease of mathematical modeling and will result in an over-estimate of the throughput.

7) Acks always make it and in zero time. (Chuckle Chuckle). The obvious inadequacies of this model are clearly being VERY generous to our physical links and are designed to "single out" the CSMA component of the throughput of our networks.

CSMA-Markov Chain Analysis

Let s_{ij} be the desired rate of transmission from Node i to Node j. Let i be a node, N_i all the neighbors of i and N_i^* be N_i and i. Let g_{ij} be the scheduled packet rates. In order to achieve s_{ij}, we must have $g_{ij} \geq s_{ij}$. Let $P(A)$ be the probability that all the nodes in a set A are not transmitting at a random instant (idle). In the steady state, $P(A)$ represents a time average. A scheduled packet from i to j will be sucessful if it finds the system in a state whereby both neighborhoods of i and j are idle. Therefore, the important quantity in determining the throughput will be $P(A)$. Since Poisson arrivals see time averages,

$$\frac{s_{ij}}{g_{ij}} = P(N_j \cup N_i) \quad (1)$$

Let g_i be the total shceduling rate out of node i and let $1/\mu_i$ be the average length of packets generated by i.

$$g_i = \sum_{j \in N_i} g_{ij} \quad (2)$$

The Markov model we choose has as its state the set of nodes transmitting. Let D be this active or busy set. D^c is the idle node set. The set D provides enough information about legal transitions which are either completions of a transmission with exponential rates μ_i for $i \in D$ or new transmission rates g_j for $j \notin D$ and $j \notin N_i, i \in D$. The latter condition results from the CSMA zero propagation delay assumption.

CSMA forces the nodes in D to be unconnected. Let N_D denote the set of all neighbors of nodes in D and D itself. Let $D + j$, $D - i$ be independent or unconnected sets formed by adding or subtracting a node from D. Assume that the network is stable for rates g_i and μ_i. Then the steady-state probabilities $Q(D)$ of states D follow the global balance equations

$$\left(\sum_{j \in D} \mu_i + \sum_{j \notin N_D} g_j\right) Q(D) = \sum_{i \in D} g_i Q(D-i) + \sum_{j \notin n_d} Q(d+j) \quad . \quad (3)$$

It follows that

$$Q(d) = \left(\pi_{i \in d} \frac{g_i}{\mu_i}\right) Q(\phi). \quad (4)$$

$Q(\phi)$ is the probability that all nodes are idle. Normalizing , this to unit mass we get

$$Q(\phi) = \left[\sum_{\text{all } D} \prod_{i \in D} \frac{g_i}{\mu_i}\right]^{-1} . \quad (5)$$

For this Markov Chain to have a steady state the null transmitter state must be positive recurrent $Q(\phi) > 0$. Finding all the independent sets D for equation 5 is an NP-complete problem. There is a very clever algorithm which allows the special structure of this formulation to be applied to networks of the size already mentioned. To find the throughput, we need the probabilities of being idle $P(A)$. Clearly this is found by summing Q(D) over all D that are contained completely in A" the compliment of A.

$$P(A) = \sum_{D \subset A^c} Q(D) \quad (6)$$

Let

$$SP(B) = \sum_{D \subset B} \left(\prod_{i \in D} g_i/\mu_i\right), \quad SP(\phi) \equiv 1. \quad (7)$$

$$P(A) = SP(A^c)/SP(V) \quad (8)$$

where V is the set of all nodes. Also

$$\frac{s_{ij}}{g_{ij}} - \frac{SP([N_i \cup N_j]^c)}{SP(V)}, j \in N_i. \quad (9)$$

We solve (9) iteratively for the g_{ij} and the throughput is identified in the steady state under this model. An analysis of the very simple network with four nodes in a linear topology is accomplished in several papers found in the bibliography in [1]. We used it to check out our program. The results are somewhat astounding. Given the four node linear topology with our convenient assumptions we get that the max throughput in the network occurs when the scheduling rate divided by the mean packet length g_1/μ_1=0.71. This maximum throughput is 0.128 packets per second. In a star topology with four legs and only five nodes in each leg the mean maximum one-way throughput was *0.054* packets per second. In a ring with 5 nodes the max throughput is 0.100 packets per second. As my final run, I took a map of EASTNET done by Zwirko, K1HTV, and under generous assumptions let the rate s_{ij} correspond to 1200 bps and assumed the mean packet length was 80 characters (a line). Using the 40 most important sites (subjective opinion) and going by word of mouth as to who had solid links, I let the algorithm crunch for a while. You are getting a maximum rate for each end to end requirement of 0.000534 packets/second. With 40*39 identical requirements (negelcting the W3IWI type choke points) this gives a network capacity of 0.833 packets per second or 9 million characters per day. This is of course awful. The most general network that can be analyzed by this technique is contained in the following

Theorem: The product form throughput evaluation and the computational procedure in Markovian CSMA networks with perfect capture and exponential packet length hold for arbitrary packet length distributions having rational Laplace transforms which need not be the same for all neighbors of a node. The analysis depends on the average normalized scheduling rate of nodes, independent of particular packet length distributions.

The bottom line is pretty clear. We have been overly generous to the network in that we neglected the real world of 0.08 bit/Hz modems run through VERY poorly equalized radios and we didn't even clobber the returning acks!! We MUST move to BTMA or some similar scheme before we grow any larger.

[1] Boorstyn, R.R., et.al, "Throughput Analysis in CSMA Packet Radio Networks", *IEEE Transactions on Communications,* Vol. COM-35, No.3, March 1987.