A Theoretical Evaluation of the G-TOR Hybrid ARQ Protocol

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ABSTRACT

The recently-introduced G-TOR protocol for HF data communications employs several features which maintain the throughput of this system in the presence of noise and interference. In this paper we take a closer look at the most important of these features - the hybrid ARQ protocol - in order to provide G-TOR users with a better understanding of the technical details of the protocol and an appreciation of the role of the Golay forward error correcting code in improving the overall system performance. We will demonstrate the advantages of using a hybrid ARQ protocol by presenting a theoretical evaluation of the throughput of the G-TOR hybrid ARQ protocol in the presence of Gaussian noise. Graphs of throughput versus channel bit error rate will show that the combined use of error detection and Golay forward error correction is a powerful approach to extending the throughput of a conventional stop-and-wait ARQ system on the HF bands.

INTRODUCTION

G-TOR (Golay-TOR) was recently introduced by Kantronics as an improved protocol for the structured interchange of digital data between stations operating in the HF Amateur bands. Simplicity was one of the design goals of this protocol - G-TOR was also developed to operate with currently-existing multi-mode **TNCs**. The system was designed to incorporate many modern digital data processing techniques. For example, **Huffman** data compression and run length encoding are used together to reduce data redundancy in each transmission. Also, fault tolerant **ACKs** and **NACKs** are employed to help prevent needless re-transmissions. The protocol is adaptive in that it allows the TNC to select a data rate based on link quality - 100, 200 or **100** baud - depending upon the number of retrys attempted. However, the most significant feature designed into G-TOR is the use of a hybrid ARQ protocol. The hybrid ARQ protocol employs forward error correction on demand. When channel conditions are good, G-TOR is simply a conventional **stop-**and-wait (S&W) ARQ system. However, when the channel deteriorates, the **Golay** forward error correction code is used in such a way that two transmissions of a frame provides enough information for the receiver to have three opportunities to reconstruct an error-free frame. The details of this process are provided in **[1]**. Also, a more detailed description of G-TOR can be found in **[2]**.

G-TOR is similar in structure to two other popular HF TOR (Teleprinting Over Radio) systems - AMTOR and **PacTOR.** All of the TOR systems are synchronous half-duplex modes which allow the exchange of digital data between two connected stations. Synchronous operation improves the efficiency of each transmission in that fewer overhead bits are required in each frame to insure bit and frame sync. G-TOR has a longer frame structure than does AMTOR and **PacTOR,** as shown in Figure 1. In any ARQ system, a short frame is likely to have fewer random errors (and hence fewer rejected frames) than in a long frame. In fact, research has shown that, because of the dynamic nature of the HF communication channel, transmissions should not be much longer than one second in duration. However, every frame must carry overhead bits which are usually independent of the frame length. Therefore, longer frames are more efficient than shorter frames in terms of the information they carry. Obviously, this is an issue which involves an engineering tradeoff for the best compromise between frame efficiency and frame length. G-TOR attacks this problem by making the frame longer to increase frame efficiency, and using interleaving to randomize and distribute the errors which may occur due to burst noise and multipath fading. The G-TOR frame structure is shown in Figure 2, and a comparison of the frame and cycle efficiencies of the HF TOR protocols is provided in Table 1.

THE GOLAY CODE

The real power of G-TOR resides in the properties of the **(24,12)** extended **Golay** forward error correcting code and the way it is used in the hybrid ARQ protocol. We decided to use the **Golay** code for the G-TOR protocol because of its simplicity and its powerful mathematical properties, most notably, the fact that it is a **half-**

rate code (i.e., the number of parity bits is the same as the number of information bits) and it is invertible. An invertible code is one in which the data bits can be recovered from the parity bits by simply running the parity bits through the Golay encoder. The half-rate feature allows the formation of parity frames which are the same length as data frames. For the interested reader, a useful tutorial on the Golay code is provided in [3].

The power of the Golay code can be seen in the following expression, which essentially gives the probability of one or more errors in a block of n-bits given that the channel bit error rate is ε:

$$\Phi = \sum_{j=0}^{t} \frac{(n)!}{(j)!\,(n-j)!} \varepsilon^{j} (1-\varepsilon)^{n-j} \qquad (1)$$

where Φ is the probability that the (24,12) Golay decoder will not be able to correct all the errors in a block of n-bits (note: n = 24 and k = 12), and t is the error correcting capability of the code The extended Golay code is capable of correcting 3 or fewer errors which may occur in any combination in a 24 bit block, so t = 3 in this case. The impact of using the Golay code is illustrated in Figure 3, which shows the improvement in error performance this code provides over an uncoded system. This is often referred to as the FEC coding *gain.*

Since the Golay code generates 12 parity bits for every 12 data bits, use of the code on every transmission would decrease the overall throughput by a factor of 1/2. This may be an acceptable tradeoff when the signal-to-noise ratio is very low, and the code is needed on every transmission to remove errors; however, in good channel conditions (high SNR) the parity bits would be an unnecessary overhead since they would seldom be needed. The solution to this problem is to use the Golay code only when it is needed, using a hybrid procedure, in which the system operates in a conventional S&W ARQ mode until errors are detected. When errors are detected, a re-transmission request from the information receiving station (IRS) results in the information sending station (ISS) transmitting parity bits instead of information bits. This procedure is summarized in the following section.

THE G-TOR HYBRID ARQ SYSTEM

An important feature of the G-TOR protocol is that it uses a type-II hybrid stop-and-wait ARQ system in combination with the 1/2-rate invertible Golay code for error correction and a 16-bit CRC code for error detection. There are two types of hybrid ARQ [4]; type I ARQ systems send both error correction and error detection parity bits with every transmission, while type II ARQ systems transmit error correction parity bits only when errors are detected in a frame. The error detection code transmitted with each G-TOR frame is a two-byte cyclic redundancy check (CRC) code. The CRC code is used to determine if the frame was received correctly before error correction is initiated; and it is also used after error correction has been completed to insure that the error correction process has successfully removed all errors in the frame.

In the type-II hybrid ARQ system, forward error correction is employed only when it is needed. The advantages of this approach can be illustrated by an example. Before transmitting a block of data to the receiver, the ISS first applies the CRC to the data for error detection, and then encodes the data using the 1/2-rate Golay FEC. The parity bits which are generated by this process are saved at the transmitter and the information bits (including the CRC) are sent. When the IRS receives the data, the check sum is computed. If the check sum passes, the data is accepted, and an ACK is returned to the transmitter. If the check sum fails, a NACK is returned to the transmitter. When the transmitter (ISS) receives an ACK, the parity bits from the first data block are discarded and the next block of data is processed. If a NACK is received, the transmitter sends the block of parity bits which had been held back. When the block of parity bits arrives at the receiver (IRS), the first action taken is to invert the parity bits to obtain data bits. Once data bits are obtained the check sum is computed. If the data (which was obtained by inverting the parity bits) passes the check, it is accepted and sent to the user. If it fails the check, then the parity bits are then combined with the data bits from the first transmission and processed by the FEC decoder. In this way, *two* concecutive transmissions provide the receiver with a total of three opportunities to obtain an error-free block of data.

EVALUATION OF HYBRID ARQ

When evaluating any data communications protocol, the most important parameter is the rate at which information (excluding overhead) is being transferred across the channel to the distant user. This is usually expressed as *throughput efficiency*, which is defined as the ratio of the average number of information bits accepted at the receiver per unit of time to the total number of bits that could be transmitted per unit of time.

Evaluating the throughput efficiency of a hybrid ARQ system is a rather difficult task. In fact, the best way to approach the problem is to consider both extremes of protocol behavior and develop an upper bound on the performance of the system [5]. Therefore, the approach taken here is to consider two situations - in the first, the protocol is assumed to be a conventional S&W ARQ system, and in the second, the protocol is assumed to use the Golay code on every alternate transmission (essentially a type I ARQ system). The true throughput behavior of the type II hybrid ARQ protocol will then be bounded by the results of both of these computations.

Since G-TOR is a hybrid version of the conventional S&W ARQ procedure, we begin by developing the throughput expression for this case. For reference, the frame timing structure for G-TOR is shown in Figures 1 and 2. Here it is assumed that M-k-bits (where k= 12 and M = # of 12-bit blocks in the frame) are transmitted at r bits/sec in a single frame followed by an interval of T-seconds during which the information receiving station (IRS) is given the opportunity to acknowledge the correct or incorrect receipt of the frame. Note that all M-k-bits in the frame do not carry information. There are a certain number, of bits devoted to overhead - frame status and error detection code parity bits for example. Therefore, we can say that there are a-bits of information in a single frame and (n – a) bits of overhead.

In one complete frame interval, the transmitter could conceivably transmit $(M \cdot k + rT)$ bits of information if it does not stay idle and if all the bits are information bits (i.e., no overhead).. These assumptions allow us to compute the true throughput efficiency of the S&W ARQ system. The throughput efficiency (defined as η), according to our definition can be interpreted as:

$$\eta = \frac{\text{Avg \# info bits received}}{\text{Max \# bits that can be transmitted}} \quad (2)$$

Where the average number of information bits received is simply $(\alpha \cdot P)$, where P is the probability that the received information will be accepted by the receiver, either because the frame arrived without errors, or because it arrived with a correctable number of errors. The throughput efficiency for the S&W ARQ system is therefore,

$$\eta = \frac{\alpha P}{M k + r T} \quad (3)$$

Where $P = ((1 - \varepsilon)^k)^M$ with ε the bit error probability of the received data. A more detailed derivation of this expression is provided in [4].

Using (3) we can evaluate the throughput performance of G-TOR as a conventional S&W ARQ system. This will provide a baseline for comparison of the performance improvement with hybrid ARQ. The various parameters used in the evaluation of (3) are provided below:

r (bits/sec)	100 bps	200 bps	300 bps
T (sec)	0.48	0.48	0.48
a (bits)	168	360	552
M·k (bits)	192	384	576
M (blocks)	16	32	48

The results of these computations are plotted in Figure 4, expressed *as throughput* which is defined in terms of througput efficiency as,

$$\text{Throughput (bits/sec)} = \eta \cdot r \quad (4)$$

Now let's include the effect of the Golay code. The Golay code significantly improves the error performance of the overall system by using the parity bits to correct up to 3 errors in a single 24-bit block. of received data. If we assume that the channel is poor and transmissions alternate between data and parity, we have established the condition for the lower bound on throughput for the system. By defining $P = \Phi^M$ in (3) and dividing by 1/2 to account for the repeated transmission, the performance of the hybrid ARQ system can be observed by plotting the throughput versus the channel bit error rate, using (3) and (4). The result is shown in Figure 5.

INTERPRETATION OF RESULTS

The essential performance of the G-TOR hybrid ARQ system can be appreciated by examining Figure 5, which evaluates the protocol at the 200 bits/sec data rate. In this graph we see that the actual information throughput (excluding overhead) is 150 bits/sec when the channel is good. As long as the received signal is strong and no interference is present, G-TOR is functioning in the S&W ARQ mode. At the opposite extreme, when the conditions are bad, G-TOR is alternating data frames and parity frames as errors are occuring in every transmission. During this time, the throughput has been cut in half because of the constant need for the parity bits.

The Golay code begins to be used frequently when the error rate is above 10^{-3}. When the channel error rate reaches approximately 2×10^{-3} the hybrid ARQ system extends the throughput and keeps it constant as the channel continues to deteriorate. The system performance is effectively extented by a factor of 10 - a considerable improvement.

CONCLUSIONS

In this paper we have briefly examined in simple form, the theoretical throughput performance of the G-TOR hybrid ARQ protocol. The performance of the system was derived by quantifying the system behavior under both poor channel and good channel conditions. In all cases we have assumed that Gaussian noise is the only interfering signal. A more realistic evaluation would need to take into account the presence of burst errors caused by manmade and natural phenomena. This type of evaulation is more reasoably accomplished through simulation or through exhaustive on-air testing. The results provided here verify that the Golay code, when used in a type II hybrid ARQ system, is an effective deterent to random bit errors.

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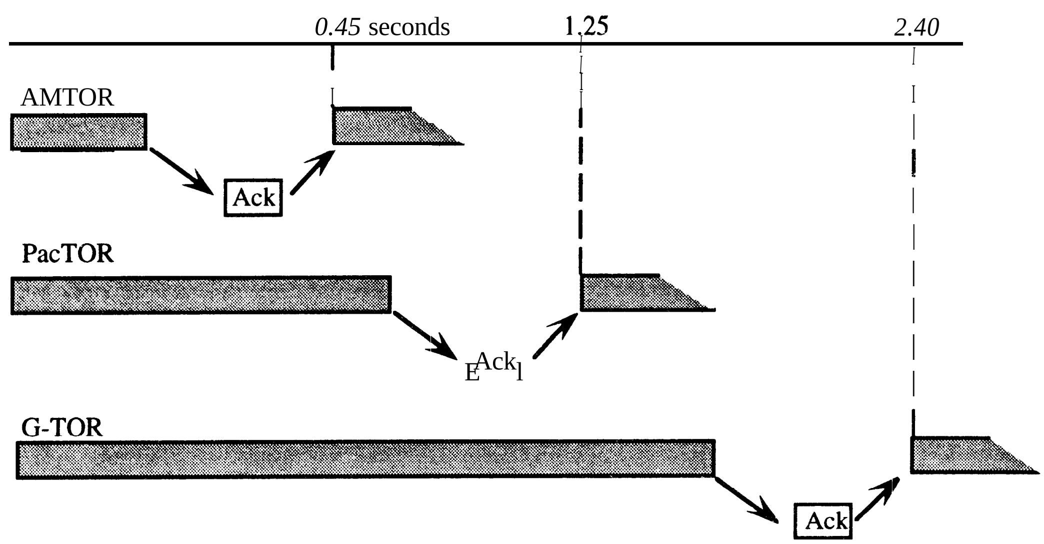

Figure 1 - TOR ARQ System Timing

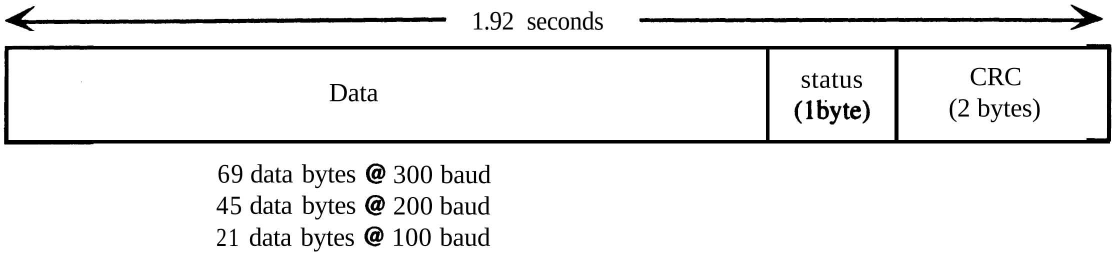

Figure 2 - G-TOR Frame Structure before Interleaving

Table 1 - HF ARQ Protocols

Mode	Baud Rate	Data (bits/frame)	Frame efficiency	Cycle efficiency	Ideal bits/sec
AMTOR	100	15	71.4%	33.33%	33.3
PacTOR-LP	100	64	66.7%	45.71%	45.7
PacTOR-LP	200	160	83.3%	57.14%	114.3
PacTOR	100	64	66.7%	5 1.20%	51.2
PacTOR	200	160	83.3%	64.00%	128.0
G-TOR	loo	168	87.5%	70.00%	70.0
G-TOR	200	360	93.8%	75.00%	150.0
G-TOR	300	552	95.8%	76.67%	230.0

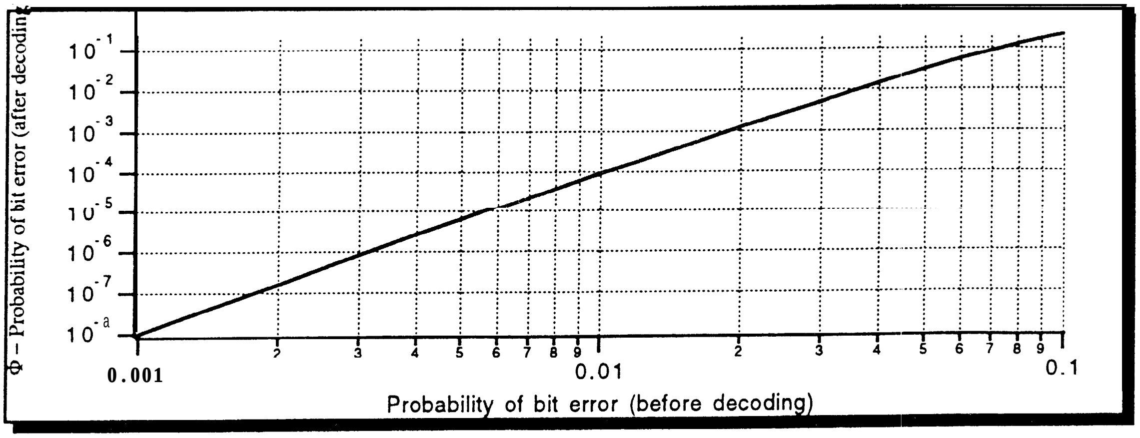

Figure 3 - Performance improvement provided by the (24,12) extended Golay code

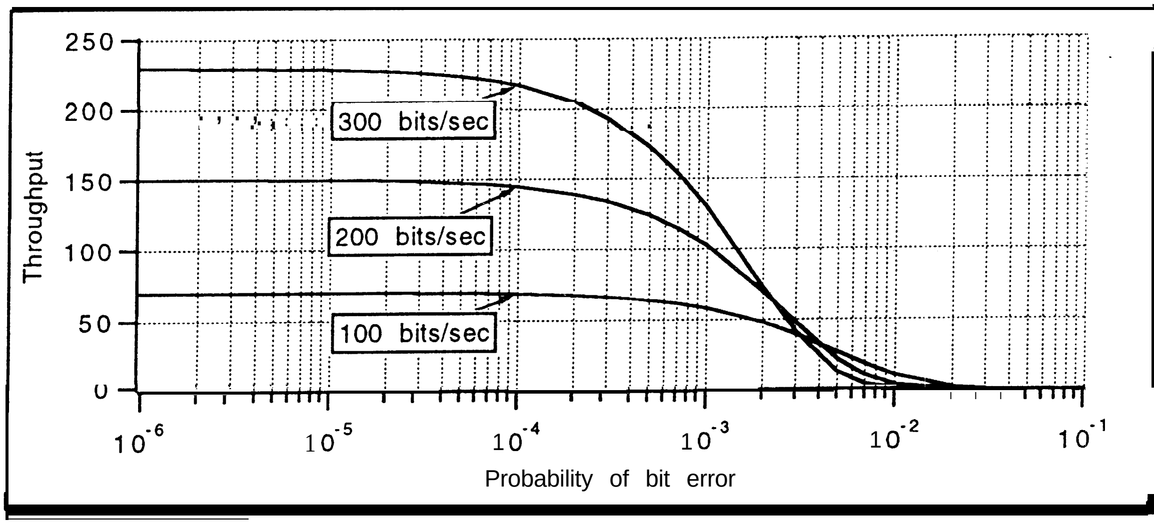

Figure 4 - S&W **ARQ** Performance of G-TOR (without the use of FEC)

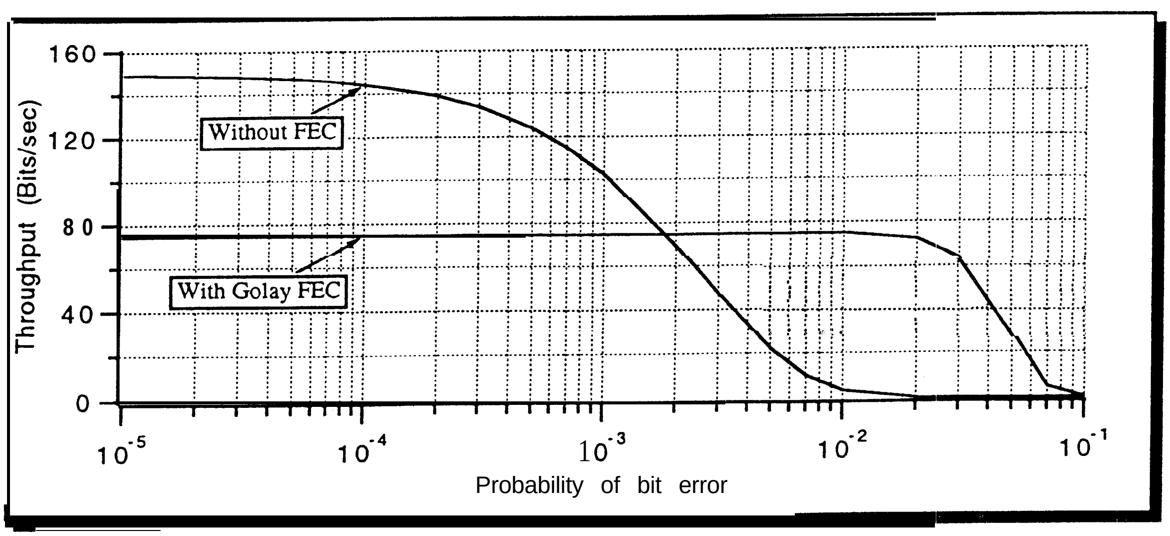

Figure 5 - Performance of G-TOR with hybrid ARQ at 200 bits/sec